# Holistic Approach to Modeling Non-Linear Internal Waves with a Three-Dimensional Nonhydrostatic Model


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## LONG-TERM GOALS

The long-term goal of this project is to develop a numerical model for the simulation of regional processes characterized by a disparity of temporal and spatial scales, such as nonlinear internal waves.

## OBJECTIVES

To develop a computational model of large amplitude internal gravity waves that uses adaptive mesh refinement in combination with a numerical scheme of order four or higher, exhibiting minimal numerical dissipation and dispersion. Furthermore, to have the capability of treating realistic topography, thus requiring the capability to describe complicated geometry and also parallel execution to reduce the large computational times intrinsic to three-dimensional simulations.

## APPROACH

### *Physical model*

We write the equations of motion as

$$\nabla \cdot \vec{u} = 0$$

$$\rho_t + (\vec{u} \cdot \nabla)\rho - N^2(1 + \beta\,\rho)v = 0$$

$$(1 + \beta\rho)\left[\vec{u}_t + (\vec{u} \cdot \nabla)\vec{u}\right] = -\nabla p - (\rho - \beta N^2 p)\vec{j} + \frac{\nu}{\rho}\nabla^2\vec{u} + \vec{F}$$

Here all quantities are assumed to nondimensionalized with reference to a wave length scale $L$, a time scale $1/N_0$ where $N_0$ is an average Brunt-Väisälä frequency and $\rho_0$ an average background

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density. We introduce the Boussinesq parameter $\beta = LN_0^2 / g$ with $g$ the gravitational acceleration. The reduced density $\rho$ is introduced such that $\beta \rho \rho$ is the density perturbation from its background stratification given by $\rho \quad y$. Similarly a reduced pressure is introduced such that $\rho \quad p$ is the pressure perturbation. The $F$ term captures external forces such as Coriolis forces and

$$\nu = \frac{\mu}{\bar{\rho}_0 L^2 N_0}$$

is the Reynolds number. In the Boussinesq approximation ($\beta \quad 0$) we set $\beta = 0$ in inertial terms but keep a finite $\beta$ in the buoyancy related terms (those exhibiting the product $N^2 \beta$).

***Numerical treatment***

**Domain discretization.** The domain of interest is simulated on logically Cartesian grids in computational space. These can be transformed into moderately complicated representations of real topography by employing grid mappings.

**Operator splitting approach**. The equations of motion are advanced in time by successively considering the various types of terms that appear. The hyperbolic terms resulting from advection

$$\rho_t + (\vec{u} \cdot \nabla)\rho = 0$$

$$\vec{u}_t + (\vec{u} \cdot \nabla)\vec{u} = \frac{\nu}{\rho} \nabla^2 \vec{u}$$

are updated first using an explicit method and with the pressure gradient approximated from the previous time step. Here it is assumed that the time step is not limited by instability of the viscous term. In this step, the quantities that are advanced in time are defined on the cell centers, whereas the advective field is defined on the cell edges.
Then the effect of source terms (gravitational acceleration and terms arising from the coordinate transformation) are added to the velocities defined on the edges of the computational cells, as well as a suitable interpolations of the fluxes calculated for the cell centered velocities

$$\vec{u}_t = -(\rho - \beta N^2 p)\vec{j} + \vec{F}.$$

Finally we perform a projection onto the space of divergence-free velocity fields to enforce the constraint

$$\nabla \cdot \vec{u} = 0$$

on the edge velocities. The potential field is then applied to the cell centered velocities as well to synchronize the two discretized versions of the same field. This operator splitting is carried out at each stage of a low-dissipation Runge-Kutta time stepping procedure.

**Grid adaptivity.** In adaptive mesh refinement one must identify regions requiring additional resolution by evaluating the local truncation error. Typically this is done through a trial step. This becomes expensive for the full equations of motion due to the effort involved in solving the Poisson equation arising from enforcing the divergence-free constraint. To avoid this, we assume that the hyperbolic terms alone give a sufficiently accurate indication of regions requiring additional resolution and let the grid adaptivity be driven by trial steps of this stage of the operator splitting.

**Poisson and Helmholtz solvers.**

Each subgrid inherits the boundary conditions for the projection scheme from the edge velocities of the parent grid already projected, so that mass conservation is always enforced. To setup and solve the necessary elliptic solvers, we have developed a package that breaks down the problem in a series of steps: The linear operators corresponding to gradient and divergence (in curvilinear coordinates) are created as a set of sparse matrices in an appropriate representation; next, the numerical laplacian is created as the matrix product of the divergence and gradient operators, ensuring consistency; finally, the resulting linear problem Ax=b is solved using existing highly efficient sparse matrix solvers. The approach is object oriented, and the resulting standalone package (ELLIKIT) can be used to deal with similar problems whenever the field to project is defined on cell edges. In particular, we have implemented hydrostatic and weakly nonhydrostatic approximations that are completely transparent to the end user.

## WORK COMPLETED

The main trust this year was devoted to implement the elliptic and parabolic solvers in the self-consistent way described above, in generalized coordinates. We also considered the generation problem of internal waves along the NJ shelf. The SW06 experiments show that arrival time at a given location across-shelf of NLIWs is not well predicted by the baroclinic tidal phase (unlike, e.g., the South China Sea case). Furthermore the largest NLIWs appear to originate during neap-tide. In fact, the barotropic tidal flow is too weak to be able to create directly NLIWs by the standard mechanism of flow over topography. In order to investigate other possible effects, we have considered what effect the presence of a shelf-break front has on the generation process. It is well known that a shelf-break front is present in the area year-around. We have thus conducted a series of experiments in which different combinations of shelf-break strength and stratification are considered.

## RESULTS

**Model**: Figure 1 shows the effect of the shelf break front on the development of the internal tide as the barotropic tide flows over the shelf edge. Here we show the density field at the end of the ebb phase. On the left, the presence of the front causes a much stronger baroclinic response near the shelf edge relative to the control case without front (right). In both cases, the barotropic tide is subcritical.

## PUBLICATIONS

A. Scotti, R. Beardsley and B. Butman, “Generation and propagation of nonlinear internal waves in Massachusetts Bay”, in press (2007).

A. Scotti, J. Wendelbo and S. Mitran, “A note on the parameterization of NLIWs in ocean models”, to be submitted to Ocean Modeling (2007).

### *Software*

The implementation of the above method are to be made publicly available to the wider research community in accordance with the grant objectives. Snapshots are periodically made available in the `/applications/2d/Boussinesq` and `/applications/3d/Boussinesq` directories of the

BEARCLAW package (http://www.amath.unc.edu/Faculty/mitran/bearclaw.html). ELLIKIT is available at ftp://ftp.unc.edu/pub/marine/ascotti

## FURTHER WORK

On the modeling side, we are currently exploring how to parameterize NLIWs within hydrostatic models such as ROMS. We are also systematically exploring the effects of the shelf-break front on the generation of NLIWs during SW06.

## FIGURES

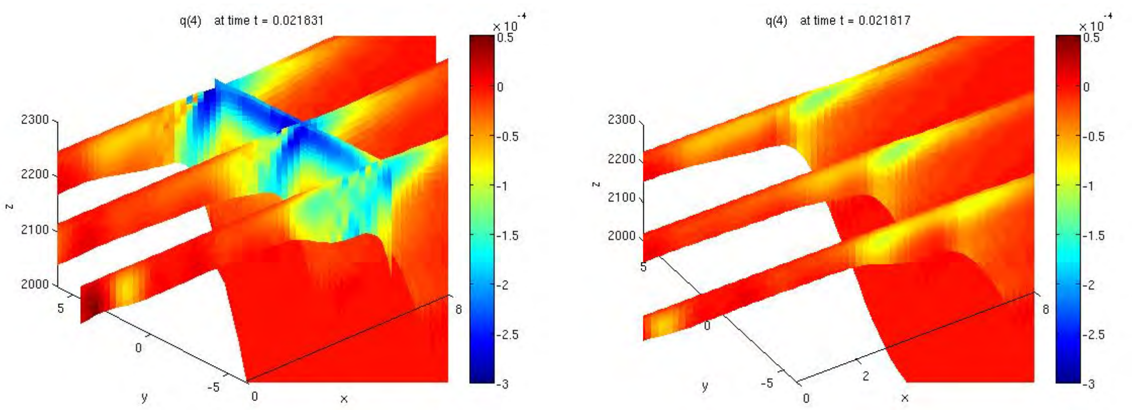


***Figure:1 [The figure shows slices of the density field near the shelf edge. One plot shows the field when the shelf-break front is present, the other without a shelfbreak front. In the former case, the baroclinic response is much stronger.]***